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Kinematics and Dynamics of a Particle in Gravitation Field

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KINEMATICSANDDYNAMICSOFAPARTICLEINGRAVITATIONFIELD

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I. Introduction

Despite its long history and the work of outstanding physicists, the general theory of relativity still contains a number of fundamental contradictions and unresolved issues. Attention is drawn to them in the monograph [1] in the chapters on the General Theory of Relativity (GR), which "is perhaps the most beautiful of all existing physical theories." In this work, we will show that all these problems are removed if, from the very beginning, changes are made to the mathematical definition of the properties of space, in which the GRelativity is described. As a sign that this is not a four-dimensional space, all of whose axes are mathematically the same, let's call it "space (3 + 1)".

II. Geometry of Space (3 + 1)

The tasks of GR mechanics can be divided into several levels: the cosmos as a whole, a galaxy, a binary star, and the last one is a point particle in a gravitational field, the scale of which is much larger than the size of the particle, and the particle's own field is much weaker than the external one. So you can consider a planet in the field of a star and a small body in the field of a planet. At each level, it can be assumed that the gravitational field is created by an object of a higher level and it can be considered stationary in those periods of time that are considered. Particle kinematics - the simplest problem of mechanics - describes the inertial world line of a point particle in the absence of any external force. For this, the differential geometry of the four-dimensional space-time is used, which takes into account the difference in the properties of time and space. The main features of this type of space follow from the formulation of the physical problem [1], but then the geometry is constructed purely mathematically. Consider the four-dimensional Euclidean space as the initial concept for describing the physical space-time. In some coordinate system, given by the starting point and four non-collinear axes, each point in space corresponds to four numbers $\left(x^0, x^1, x^2, x^3\right)$ that are the coordinates of this point. To use the apparatus of differential geometry, it is necessary to pass to curvilinear coordinates. To do this, we introduce four linearly independent continuous differentiable functions $x'^i = f_i\left(x^0, x^1, x^2, x^3\right)$. This coordinate transformation must be reversible and differentiable. At each point *M* of space, four coordinate lines intersect. They are determined by a change in one of the coordinates with fixed values of the other three corresponding to the point *M*. The tangents to these lines are linearly independent and form a local frame. Any vector drawn from the point *M* can be decomposed with respect to this frame. Now the same Euclidean space is described in curvilinear coordinates. The transition to curvilinear coordinates is unambiguous and reversible. This is due to the fact that the Euclidean space is affine. In addition, in the geometry of the physical space-time, three spatial coordinates must be equivalent, and the time coordinate must differ in special properties. Therefore, we will call such a space not four-dimensional, but the space (3 + 1). Special Theory of Relativity (SR) is not described in four-dimensional Euclidean space, but in (3+1) Minkowski space, which is called "index one pseudo-Euclidean space". In this space, the time coordinate is a purely imaginary number. This made it possible to simply describe all the differences between the mechanics of the SR and classical mechanics. But with such a choice of one of the coordinate axes, the transition to curvilinear coordinates described above would lead to the fact that all coordinates x'^i become complex numbers and the selection of the time coordinate disappears.

The properties of the space (3+1) can be described by the properties of the metric tensor of this space. This possibility has long been used in describing the kinematics of the GR, but in well-known monographs

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on this theory, the description of the properties of the metric tensor is not presented systematically.

A multidimensional space is called metric if the scalar product of vectors $(\boldsymbol{A}\cdot\boldsymbol{B}) = G_{ik}A^iB^k$ is defined in it. Here $\boldsymbol{A},\boldsymbol{B}$ are the vectors plotted from one point in space, $A^i, B^k -$ their components in the local frame of this point, $G_{ik} -$ are the components of the symmetric tensor. (Hereinafter, the usual rule of summation over repeated indices is adopted. Indexes denoted in Latin letters take the values 0, 1, 2, 3; indexes denoted in Greek letters take the values 1, 2, 3). The metric tensor $\|\boldsymbol{G}\|$ is a rank 4 symmetric non-singular matrix whose components generally depend on the coordinates. Like any real symmetric non-singular matrix, the metric tensor at each point in space can be reduced to a diagonal form, that is, there is such a real non-singular matrix $\|\boldsymbol{D}\|$ that the matrix $\|\boldsymbol{D}\|^{-1}\cdot\|\boldsymbol{G}\|\cdot\|\boldsymbol{D}\| = \|\bar{\boldsymbol{G}}\|$ is diagonal (this is indicated by a bar over the matrix character). If the components of the metric tensor depend on the coordinates, then its diagonal components also depend on the coordinates.

The matrix can also be reduced to a diagonal form by introducing a rectilinear orthogonal coordinate system at the selected point, which is called Galilean. Such a transformation reveals properties that are important for what follows. At the origin, located at a chosen point in space, generally speaking, inhomogeneous, the metric tensor will be exactly diagonal, as in the case of an algebraic transformation of the matrix but for small deviations from the origin, the off-diagonal components will be of the second order of smallness in deviations. If the space is Euclidean, then the metric tensor does not depend on the coordinates and can be reduced to a diagonal form by introducing Galilean coordinates throughout the space.

The matrix $\|\boldsymbol{D}\|$ or orientation of the Galilean coordinates frame can be chosen in an infinite number of ways. In this case, the metric tensor of the space (3+1) SR always has the form {see [1] formula (6.5)}:

$$\|\boldsymbol{G}_0\| = \begin{Vmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{Vmatrix}. \quad (2.1)$$

Further we will denote the metric tensor $\|\mathbf{G}\|$ and its components $\mathbf{g}_{ij}$.

Let us describe the difference in the physical meaning of the zero row and the remaining three rows of the metric tensor matrix. The theory should describe not the trajectory of a particle in three-dimensional space, but the world line in four-dimensional space-time (3+1). General view of the world line:

$$s\left\{x^0 = ct, x^1(t), x^2(t), x^3(t)\right\}. \quad (2.2)$$

Here $t-$ time, a continuously changing parameter $dt > 0$, $x^0 -$ a coordinate proportional to time with the coefficient $c -$ of the electromagnetic wave velocity in vacuum, the remaining variables $x^\alpha(t)$ are coordinates of a point in a certain system of curvilinear coordinates in a curved three-dimensional space.

The metric tensor of a homogeneous space (3+1) in Galilean coordinates $\|\bar{\mathbf{G}}_0\|$ is defined by the equality

$$\mathbf{g}_{ij} = \delta_{ij}\left(2\delta_{i0} - 1\right). \quad (2.3)$$

Let us establish the rule that if the matrix is diagonal, then the rows whose diagonal element is positive are located in the upper part of the matrix. This rule was introduced in the monograph [1] and is called the signature. In the described case, the diagonal element is positive in the top row, and are negative in the remaining rows. This defines an important property of the space: a sequence of signs in an invariant quadratic form that describe the curve arc differential:

$$ds^2 = \left(dx^0\right)^2 - \left(dx^1\right)^2 - \left(dx^2\right)^2 - \left(dx^3\right)^2. \quad (2.4)$$

The main axiom of the GR: in a gravitational field, it is possible to bring the metric tensor to a diagonal form only locally at each point in space. This geometry is called Riemannian geometry. The fundamental Einstein equation relates the curvature of space to the distribution and movement of mass in space. We consider a stationary gravitational field. Therefore, the energy-momentum tensor does not depend on time, and the zero value of the indices does not make sense. The geometry of three-dimensional space is determined by the Einstein equations, in which the indices take only the values 1, 2, 3.

$$\mathbf{R}_{\alpha\beta} - \frac{1}{2}\mathbf{g}_{\alpha\beta}R = \frac{8\pi k}{c^4}\mathbf{T}_{\alpha\beta}. \quad (2.5)$$

Here the constant k is the gravitational constant, $\mathbf{T}_{\alpha\beta}$ are components of the energy-momentum tensor of the mass that creates the gravitational field, the Ricci tensor $\|\mathbf{R}\|$ and its convolution R (scalar curvature) are expressed in terms of the metric tensor of three-dimensional space and its derivatives with respect to coordinates. If the space is flat, then the Ricci tensor is identically equal to zero.

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Then, obviously, the energy-momentum tensor is also equal to zero. The converse is also true: if the energy-momentum tensor is identically equal to zero, then the space is flat, i.e., Euclidean.

Einstein's equations define the matrix of the metric tensor of the three-dimensional Riemannian space $\left\| \mathbf{g}_{\mu\nu}(\boldsymbol{r}) \right\|$, components of this matrix are functions of the point at which the metric is defined. The metric space tensor (3+1) can be represented by a cellular matrix

$$\left\| \mathbf{G} \right\| = \left\| \begin{matrix} \mathbf{g}_{00}(\boldsymbol{r}) & \mathbf{g}_{0\alpha} \equiv 0 \\ \mathbf{g}_{\alpha 0} \equiv 0 & \left\| \mathbf{g}_{\mu\nu}(\boldsymbol{r}) \right\| \end{matrix} \right\|. \qquad (2.6)$$

This matrix must be cell-diagonal. This means that three-dimensional space is a hypersurface in four-dimensional space on which a Riemannian metric is defined.

This definition of the metric tensor in (3+1) space is the main new mathematical proposition in this paper. Further, we will show that if it is accepted, the entire further theory of the motion of a particle in a gravitational field is built logically sequentially. All the problems noted but not resolved in the monograph [1] disappear, and the limiting transition to the special theory of relativity becomes clear. In [1] the metric tensor of three-dimensional space was also introduced, but it was obtained as a result of a thought experiment, which is incorrect for geometry. As a result the formula expressing its components in terms of the components of the metric tensor of the four-dimensional space is incorrect. The identity $\mathbf{g}_{0\alpha} \equiv 0$ means that all components in this row and column, except for the diagonal one, are equal to zero; $\mathbf{g}_{00} = \mathbf{g}(\boldsymbol{r})$ is a scalar function of spatial coordinates. The coordinate $\boldsymbol{r}$ is a three-dimensional space vector; in contrast to 4-vectors, we will denote them by small letters.

Let's move on to Galilean coordinates at a fixed spatial point $\boldsymbol{r}$ and reduce to diagonal form the matrix $\left\| \mathbf{G}(\boldsymbol{r}) \right\|$. To reduce to diagonal form a cell matrix of the fourth rank, it is necessary to reduce to diagonal form only the matrix of the third rank $\left\| \mathbf{g}_{\mu\nu}(\boldsymbol{r}) \right\|$. The metric tensor of three-dimensional space defines by Einstein's equation (2.5) for three-dimensional space. This is a matrix of the third rank with components $\mathbf{g}_{\alpha\beta}$. The characteristic equation of such a matrix is the equation of the third degree. Therefore, it has one, two or three real roots. The components of the metric tensor $\left\| \mathbf{g}_{\mu\nu}(\boldsymbol{r}) \right\|$ reduced to a diagonal form in Galilean coordinates must be equal to each other. The metric tensor of a three-dimensional space, reduced to a diagonal form at a point $\boldsymbol{r}$, should have the form:

$$\left\| \mathbf{g}_{\mu\nu}(\boldsymbol{r}) \right\| = g(\boldsymbol{r}) \left\| \begin{matrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{matrix} \right\|, \qquad (2.7)$$

where $g(\boldsymbol{r})$ is a real function of spatial coordinates. That is the real root of the cubic characteristic equation of the matrix $\left\| \mathbf{g}_{\mu\nu}(\boldsymbol{r}) \right\|$:

$$y^3 - ay^2 + by - c = 0$$

$$a = \boldsymbol{g}_{11} + \boldsymbol{g}_{22} + \boldsymbol{g}_{33}, \quad b = \left[\boldsymbol{g}_{12}{}^2 + \boldsymbol{g}_{13}{}^2 + \boldsymbol{g}_{23}{}^2 - \boldsymbol{g}_{11}\boldsymbol{g}_{22} - \boldsymbol{g}_{33}\boldsymbol{g}_{22} - \boldsymbol{g}_{33}\boldsymbol{g}_{11} \right]. \qquad (2.8)$$

$$c = \boldsymbol{g}_{11}\boldsymbol{g}_{22}\boldsymbol{g}_{33} - \left[\boldsymbol{g}_{11}\boldsymbol{g}_{23}{}^2 + \boldsymbol{g}_{22}\boldsymbol{g}_{13}{}^2 + \boldsymbol{g}_{33}\boldsymbol{g}_{21}{}^2 \right] + 2\boldsymbol{g}_{12}\boldsymbol{g}_{23}\boldsymbol{g}_{31}$$

If there are two or three such roots, then one can choose a single value based on physical considerations. For example, if space should tend to flatten as it moves away from a certain point or area, then the function $g(\boldsymbol{r})$ should tend to unity. The function $g(\boldsymbol{r})$ must be positive. The equation $g(\boldsymbol{r}) = 0$ defines a surface in space separating the accessible $g(\boldsymbol{r}) > 0$ and inaccessible $g(\boldsymbol{r}) < 0$ regions of space, and the inaccessible region must be limited if the mass that creates the energy-momentum tensor that determines the gravitational field in Einstein's equations occupies some limited volume. For example, in the case of spherical symmetry the function $g(\boldsymbol{r})$ must be proportional to $(r - r_0)/r$, where r_0 is the radius of the Schwarzschild sphere.

Let $\boldsymbol{g}_{00}(\boldsymbol{r}) = g(\boldsymbol{r})$. Let us emphasize that this condition is necessary for the cell matrix to be proportional to the metric tensor (2.3). Then formula (2.6) can be represented as:

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$$\|\mathbf{G}\| = \begin{Vmatrix} g(\boldsymbol{r}) & \mathbf{g}_{0\alpha} \equiv 0 \\ \mathbf{g}_{0\alpha} \equiv 0 & \|\mathbf{G}_S\| \end{Vmatrix} \equiv g(\boldsymbol{r}) \begin{Vmatrix} 1 & \mathbf{g}_{0\alpha} \equiv 0 \\ \mathbf{g}_{0\alpha} \equiv 0 & \|\mathbf{G}_S\| / g(\boldsymbol{r}) \end{Vmatrix}. \quad (2.9)$$

Total metric space tensor (3+1) in Galilean coordinates at each point $\boldsymbol{r}$:

$$\|\boldsymbol{G}\| = g(\boldsymbol{r}) \begin{Vmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{Vmatrix}. \quad (2.10)$$

We will not present differential geometry. It is described in sufficient detail in the monograph [1]: §83, §85, §86. We give only some definitions and formulas necessary for further presentation.

The space (3+1) is a special case of a four-dimensional space with a metric tensor defined by us. Therefore, general concepts and formulas can be used, taking into account formula (2.9). The curve in the space (3+1) is generally given in the form $s\left[x^0 = ct, x^1(t), x^2(t), x^3(t)\right]$. The tangent vector to this curve $\boldsymbol{U}$ is defined by the equations (see [1, §7]):

$$ds = dt\sqrt{c^2 - (v^1)^2 + (v^2)^2 + (v^3)^2} = cdt\sqrt{1 - |\mathbf{v}|^2/c^2};\ v^\alpha = \frac{dx^\alpha}{dt};$$

$$U^i = dx^i/ds;\ U^0 = c\frac{dt}{ds} = \frac{1}{\sqrt{1-|\mathbf{v}|^2/c^2}},\ U^\alpha = \frac{v^\alpha}{c\sqrt{1-|\mathbf{v}|^2/c^2}},\quad |\boldsymbol{U}|^2 = 1. \quad (2.11)$$

The general definition of a geodesic line is a line whose tangents are parallel at any point. In addition, the length of the tangent segment, given at one, arbitrarily chosen point, is preserved in the Riemannian space. Through any point in space in any direction it is possible to draw a geodesic and only one. In different sources, you can find different definitions of the equations of a geodesic line. Usually they are represented as the equality to zero of the sum of the derivative of the corresponding component of the vector $\boldsymbol{U}$ along the length of the arc and the term proportional to the connection coefficient, which is determined by a linear combination of the derivatives of the metric tensor with respect to the coordinates.

For our purposes, it is more convenient to use the form of differential equations of a geodesic line presented in the monograph [2]:

$$\frac{dU_i}{ds} - \frac{1}{2}\frac{\partial g_{kl}}{\partial x^i}U^k U^l = 0. \quad (2.12)$$

Since we know the diagonal form of the matrix in this equality, using this, we get:

$$\frac{dU_i}{ds} = \frac{\partial g(\boldsymbol{r})}{\partial x^i}|\boldsymbol{U}|^2 = \frac{\partial g(\boldsymbol{r})}{\partial x^i}. \quad (2.13)$$

III. Kinematics and Dynamics of a Particle in Space (3+1)

The main task of mechanics in the macrocosm is to determine the world lines of particles in various conditions. In pseudo-Euclidean space, in the absence of forces, it is always a straight line, determined by the initial conditions: the starting point and the velocity vector. This is formulated in Newton's first law - the law of inertia: "A particle maintains a state of rest or uniform, rectilinear motion until an external force acts on it." In the Riemannian space in this law, only the words "... uniform, rectilinear ..." should be replaced by "... movement at a constant speed." In Euclidean space, these expressions are equivalent, the velocity vector can be moved along a straight line. In a Riemannian space, the parallel translation of a tangent vector from a certain starting point occurs along a geodesic line.

Let us pass in equations (2.13) from geometric quantities to physical ones, that is, observable, measurable and having dimensions. This is a system of four equations that differ only in the index value. But in the space (3 + 1), the equation for the time axis ($i = 0$) has some differences due to the fact that the function $g(\boldsymbol{r}(t))$ depends on time only through the dependence of coordinates on time, i.e. due to the motion of the particle. Then we get:

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$$\frac{dU_0}{ds}=\frac{\partial g}{\partial x^\alpha}\frac{v^\alpha}{c}\left(U^0\right)^2-3\frac{\partial g}{\partial x^\alpha}\frac{v^\alpha}{c}\left(U^\mu\right)^2=\frac{\partial g}{\partial x^\alpha}\frac{v^\alpha}{c}\left|U\right|^2=\frac{\partial g}{\partial x^\alpha}\frac{v^\alpha}{c}$$

$$\frac{dU_\alpha}{ds}=-\frac{\partial g}{\partial x^\alpha}\left(U^0\right)^2+3\frac{\partial g}{\partial x^\alpha}\left(U^\mu\right)^2=-\frac{\partial g}{\partial x^\alpha}\left|U\right|^2=-\frac{\partial g}{\partial x^\alpha} \qquad (3.1)$$

On the right side of the equalities, we pass to differentiation with respect to time using formulas (2.11). We obtain equations expressed in terms of coordinates and time:

$$\text{a)} \qquad \frac{dU_0}{ds}=\frac{d}{dt}\left(\frac{1}{\sqrt{c^2-\boldsymbol{v}^2}}\right)=0=\frac{\partial g}{\partial x^\alpha}\frac{v^\alpha}{c}$$

$$\text{b)} \qquad \frac{dU^\alpha}{ds}=\frac{d}{dt}\left(\frac{v^\alpha}{c\sqrt{1-\boldsymbol{v}^2/c^2}}\right)=\frac{1}{c\sqrt{1-\boldsymbol{v}^2/c^2}}\frac{dv^\alpha}{dt}=-\frac{\partial g}{\partial x^\alpha}. \qquad (3.2)$$

$$\text{c)} \qquad \frac{d}{dt}\left(\frac{E\left(\boldsymbol{v}^2\right)}{c}\right)=0, \quad \frac{dp^\alpha}{dt}=-\frac{\partial\left(mcg\right)}{\partial x^\alpha},$$

Here, as in formulas (2.11), $v^\alpha=dx^\alpha/dt$ - spatial velocity, $E\left(\boldsymbol{v}^2\right)$ is a kinetic energy. If we multiply the equations by mc, where m is the mass of the particle, then the function $mcg\left(\boldsymbol{r}\right)$ plays the role of a potential. Then it follows from equations a) and b) that in three-dimensional space the gravitational force is perpendicular to the velocity. Therefore, this potential affects the form of the trajectory, but does not change the velocity modulus, that is, the kinetic energy. This is a well-known natural phenomenon: under the influence of gravity, planets move in closed orbits without changing their kinetic energy. In our theory, this happened due to the fact that it was accepted $\boldsymbol{g}_{00}\left(\boldsymbol{r}\right)=g\left(\boldsymbol{r}\right)$.

Equations expressed in terms of impulses are written in line c). As is known (see [1]), kinetic energy can be expressed in terms of spatial impulses and rest energy. This function is called the Hamiltonian function $H=c\sqrt{m^2c^2+\boldsymbol{p}^2}$ and the equation for the zero momentum component is:

$$\frac{d}{dt}H=0 \;. \qquad (3.3)$$

This is the law of conservation of energy in a gravitational field. The remaining three equations describe Newton's second law: acceleration is proportional to the acting force.

The monograph [3] formulates the rules for projecting a Riemannian space onto a flat one. With this transformation, the geodesic line becomes a straight line. This is obvious, since the defining property of a geodesic is the constancy of direction. Then the opposite is also possible: the transformation of a straight line into a geodesic. These transformations open up the possibility of transition to another frame of reference through Lorentz transformations.

In the monograph [1] in the footnote to § 85: "It can be shown that by a suitable choice of the coordinate system it is possible to achieve the vanishing of all connection coefficients not only at one point, but also throughout the given world line." (The proof of this statement can be found in P. K. Rashevsky's book "Riemannian Geometry and Tensor Analysis". Nauka, (1964), §91.)". The solution of the above specific problem is a special case of the theorem proved in the monograph [3], therefore, using this theorem, one can consider the problem of the combined action of gravitational and electromagnetic fields.

IV. Conclusion

The equations of motion of a particle in a gravitational field are obtained by sequentially taking into account the position that the physical four-dimensional space is a distinguished time axis and a three-dimensional hypersurface, the metric tensor of which is determined by the Einstein equations. This approach removes all the problems and contradictions noted in the monograph [1], and the resulting equations adequately describe, for example, the curvilinear motion of planets without energy loss.

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